Higher Maths Essential Skills

Mastering the skills form the National Maths 5 course is essential if you are to make a smooth transition to the Higher Maths course.

You must do the following work and mark it before returning to school after the summer holidays. If you have gaps in your understanding, you must attend study support to address this.

Video help for each exercise is available by searching at https://www.youtube.com/user/MrYoungsMaths/playlists then selecting the 'SQA Higher Maths Essential Skills' playlist.

Adding and subtracting algebraic terms

Algebraic expressions normally follow these conventions:

- terms are ordered according to their powers, usually with the highest power first
- variables within terms are put in alphabetical order
- where possible a negative first term is avoided.

To simplify algebraic expressions we can combine like terms.

Example 1

Simplify $3a^2 + 5ab + 2b^2 - 3b + 4ba + 7a^2$.

$$3a^2 + 5ab + 2b^2 - 3b + 4ba + 7a^2 = 3a^2 + 7a^2 + 5ab + 4ab + 2b^2 - 3b$$
$$= 10a^2 + 9ab + 2b^2 - 3b$$

Exercise 1

1 Simplify where possible:

(a) $3pq + 5pq$
(b) $7xy + 2zy$
(c) $km + 6mk$
(d) $12cd^2 - 5cd^2$
(e) $13rs^2 - 8r^2s$
(f) $7ax^2 + 3bx^2$
(g) $4v^2w + 3wv^2$
(h) $9p^2q^2 - q^2p^2$
(i) $3y^2 - 2y^2 + 7y - y$
(j) $r^3 + 2r^3 - 8r + 5r$
(k) $b^2 + 3b^2 - ab - 2ab$
(l) $5x^2 + x^2 - x + x$
(m) $ab + 3xy + 5ab$
(n) $pq - kl + 7pq$
(o) $8xy^2 + xy - 6xy^2$
(p) $9vw^3 - 7v^3w - vw^2$
(q) $13cd - 20cd + dc$
(r) $a^3 + b^3 - a^3b^3$
(s) $2x^3 + 3x^2 - x^3$
(t) $5x^2 + x^3 - 9x^2$

2 Simplify:

(a) $x^2 + 10 - x^2 + 10$
(b) $p^2 + p + p^2 + 2p$
(c) $m^2 + m + 1 - m^2 - 2m - 3$
(d) $-3b^2 + 2b + 4 + b^2 - 2b + 4$
(e) $a - 2c + 3b - 2b + a + 5c$
(f) $-p - r - q + 7p - 4q - r$
(g) $4x^2 - 5xy - 2xy - x^2$
(h) $9s^2 - 5r^2 - 8r^2s^2 - 8s^2$
(i) $2.7k - 1.3j + 3.3k + 0.9j$
(j) $\frac{1}{4}t^2 - s + \frac{1}{2} + \frac{1}{2}t^2 - \frac{1}{2}s + \frac{1}{4}$
(k) $4x^2 - 8y^2 - 3xy + 5xy - 2y^2$
(l) $ab + a^2b - 9ab + 5a^2b + b^2a$

Multiplying and dividing algebraic expressions

We can also multiply and divide algebraic expressions.

Example 2

Simplify

(a) $(3p)^2 \times 5kp$ (b) $24x^2y \div 8y^2x$

(c) $-9t(st - 5t^2)$ (d) $\dfrac{20x^2 - 5xy}{10x}$

(a) $(3p)^2 \times 5kp = 9p^2 \times 5kp$
$= 9 \times p^2 \times 5 \times k \times p$
$= 45 \times p^2 \times k \times p$
$= 45 \times p^3 \times k$
$= 45kp^3$

(b) $24x^2y \div 8y^2x = \dfrac{24x^2y}{8xy^2}$
$= \dfrac{3x}{y}$

(c) $-9t(st - 5t^2) = -9st^2 + 45t^3$
$= 45t^3 - 9st^2$

(d) $\dfrac{20x^2 - 5xy}{10x} = \dfrac{20x^2y}{10x} - \dfrac{5xy}{10x}$
$= 2xy - \dfrac{y}{2}$

Exercise 2

1 Simplify:

(a) $7a \times 3a^2b$ (b) $-8xy \times 4xy$ (c) $mn^2 \times m^2n$

(d) $ab \times pq$ (e) $-k \times (-km^2)$ (f) $cd \times (-d^2)$

(g) $\dfrac{1}{a} \times a$ (h) $r^2 \times \dfrac{p}{r}$ (i) $\dfrac{v}{w^2} \times \dfrac{w}{v}$

(j) $(2b)^2 \times 3b$ (k) $5e \times (5f)^2$ (l) $(-g)^2 \times 7fg$

(m) $(7x)^2 \times (-x)^2$ (n) $-4p \times (pqg)^2$ (o) $\left(\dfrac{1}{a}\right)^3 \times a^2$

(p) $\dfrac{3}{(2s)^2} \times 8rs$ (q) $\dfrac{1}{x^3} \times x^3y$ (r) $\dfrac{a}{b^2} \times \dfrac{b}{a^2}$

2 Simplify:

(a) $6ab \div 3a$ (b) $15xy \div 5y$ (c) $36cd \div 9cd$

(d) $7pq^2 \div pq$ (e) $20rs^2 \div 4s^2$ (f) $54f^2g \div 9f$

(g) $12ab^2 \div 4a^2b$ (h) $16uv \div 8uv^2$ (i) $19ab \div ab^2$

(j) $3m^2n \div 18mn$ (k) $7gh^2 \div 14h^3$ (l) $28k^2l^2 \div 7k^2l$

(m) $3tu \div 39t^2u^2$ (n) $33vw^3 \div 11vw$ (o) $42xy^2 \div 6x^2y$

3 Expand:

(a) $3x(5x + 2)$ **(b)** $7y(y^2 - z)$ **(c)** $-3a^2(2 - 5a^2)$

(d) $-(9p - q)$ **(e)** $12r(s^2 + r)$ **(f)** $-w^2(vw + 1)$

(g) $(13x - 5y)3y$ **(h)** $(a^2 + b^2)abc$ **(i)** $(-5d^3 - e^2)e$

(j) $f^2g(3g - 2f)$ **(k)** $(m^2 + 5n)km$ **(l)** $-8t^3(u - tu)$

4 Simplify:

(a) $\dfrac{27x + 18}{3}$ **(b)** $\dfrac{35x^2 - 5y}{5}$ **(c)** $\dfrac{x^2 - 5x}{x}$

(d) $\dfrac{12ab + a^2}{4a}$ **(e)** $\dfrac{48p^2q - 16pq}{8p}$ **(f)** $\dfrac{5r^2s^2 + 7rs^2}{rs}$

(g) $\dfrac{pqr + p^2qr^2}{pqr}$ **(h)** $\dfrac{14x^3y^2 - 28xy + 7x^2y}{7xy}$

Expanding brackets

We can simplify algebraic expressions by expanding terms in brackets.

Example 3

Expand the following expressions:

(a) $(3x - 4) - 6x - 2(3 - x)^2$ **(b)** $(2x + 3)(-5x + 3)$ **(c)** $(2x - 5)^3$

(a) $x^2(3x - 4) - 6x - 2(3 - x)^2$

$= x^2(3x - 4) - 6x - 2(9 - 6x + x^2)$ _____ expand squared bracket

$= 3x^3 - 4x^2 - 6x - 18 + 12x - 2x^2$ _____ remove brackets

$= 3x^3 - 6x^2 + 6x - 18$ _____ simplify

(b) $(2x + 3)(x^2 - 5x + 3)$

$= 2x(x^2 - 5x + 3) + 3(x^2 - 5x + 3)$ _____ multiply terms in the second bracket

$= 2x^3 - 10x^2 + 6x + 3x^2 - 15x + 9$ _____ by each term from the first

$= 2x^3 - 7x^2 - 9x + 9$ _____ simplify

(c) $(2x - 5)^3 = (2x - 5)(2x - 5)(2x - 5)$

$= (2x - 5)(4x^2 - 20x + 25)$ _____ multiply a pair of brackets

$= 2x(4x^2 - 20x + 25) - 5(4x^2 - 20x + 25)$ _____ multiply second bracket by

$= 8x^3 - 40x^2 + 50x - 20x^2 + 100x - 125$ _____ each term from the first

$= 8x^3 - 60x^2 + 150x - 125$ _____ simplify

Exercise 3

1 Expand:

(a) $(x + 3)(x + 4)$
(b) $(4 - y)(6 - y)$
(c) $(p - 2)(p - 13)$
(d) $(r + 5)(r - 6)$
(e) $(4 - s)(8 + s)$
(f) $(2t + 5)(t - 7)$
(g) $(3v - 1)(7v + 4)$
(h) $(2 - 9w)(4 - 3w)$
(i) $2(x + 5)(x + 7)$
(j) $5(a + 2)(a - 9)$
(k) $7(1 - t)(4 - t)$
(l) $-4(m + 2)(m - 10)$

2 Expand:

(a) $(x + 2)^2$
(b) $(3y - 5)^2$
(c) $(7 + 2a)^2$
(d) $(9 - 4q)^2$
(e) $(x + 2)(x - 2)$
(f) $(3k - 1)(3k + 1)$
(g) $(20 + 7q)(20 - 7q)$
(h) $3(x + 1)(x - 1)$
(i) $5(a - 2)(a + 2)$
(j) $10(3 + p)(3 - p)$
(k) $6(m - n)(m + n)$
(l) $2(3k + 4j)(3k - 4j)$
(m) $(b + 5)^2$
(n) $(d - 6)^2$
(o) $(8 - k)^2$
(p) $(10 + p)^2$
(q) $(9 + 2s)^2$
(r) $(7 - 5v)^2$
(s) $(3p - q)^2$
(t) $(5m + 2n)^2$
(u) $(a + b)^2$
(v) $(g - 7h)^2$
(w) $(5x - 8y)^2$
(x) $(6v + 9w)^2$

3 Expand and simplify:

(a) $x(2x + 4) + 3x - 5(x^2 - 6)$
(b) $x^2(5x - 7) + 4x(3x + 2) - x^2$
(c) $8x(5x - 6) - 3(x + 4)^2$
(d) $y^2(y + 5) - 7y - (y - 5)^2$
(e) $9(a - 6)^2 - a(2a + 3)^2$
(f) $25p^2 - (5 - p)^2 + 10p$
(g) $4f(3f - 5)^2 + 20f - 2(3f - 1)^2$
(h) $t^2(9t - 1) - 9(t + 1)^2 + 10t^2$
(i) $-6s(7 + s)^2 + (2s - 6)^2$
(j) $(8 - 2w)^2 - 60 + 10w - (w - 4)^2$

4 Expand and simplify:

(a) $(x + 1)(x^2 - 2x + 3)$
(b) $(2y + 4)(y^2 + 5y - 6)$
(c) $(5a - 2)(3a^2 - 7a + 4)$
(d) $(b^2 + 5b - 2)(6b - 1)$
(e) $(7p^2 - 8p - 9)(3p + 6)$
(f) $(1 - q)(5 - 2q + q^2)$
(g) $(8 + t)(8 - 3t + 3t^2)$
(h) $(6 - 4s - 2s^2)(9 - 5s)$
(i) $(2 + x)(3x^2 + 5x + 1)$
(j) $(6 - d)(7d^2 + 2d - 8)$
(k) $(f + 9)(5 - 7f + 4f^2)$
(l) $(2 - 4h - 3h^2)(6h + 9)$
(m) $2(w + 5)(w^2 - 3w + 1)$
(n) $3(z^2 + 7z - 2)(2z - 1)$
(o) $5(2 - a)(3a^2 + a - 2)$
(p) $4(1 + b)(7 - 2b - b^2)$

5 Expand and simplify:

(a) $(k + 4)^3$ **(b)** $(3r - 5)^3$

(c) $(p - 1)^3$ **(d)** $(4 + 2q)^3$

(e) $(a + 2)^3$ **(f)** $(b - 1)^3$

(g) $(2d + 3)^3$ **(h)** $(5e - 2)^3$

(i) $(1 + 4x)^3$

Factorising

Some algebraic expressions can be written as a product of factors. This is called factorisation.

Example 4

Factorise fully:

(a) $3ab^2 + 9ab$ **(b)** $x^2 + 5x + 6$ **(c)** $20 - 15a - 5a^2$

(d) $y^2 - 81$ **(e)** $72 - 2t^2$ **(f)** $t^4 - 1$

(a) $3ab^2 + 9ab = 3ab(b + 3)$ — common factor

(b) $x^2 + 5x + 6 = (x + 2)(x + 3)$ — quadratic factors

(c) $20 - 15a - 5a^2 = 5(4 - 3a - a^2)$ — common factor then
$= 5(4 + a)(1 - a)$ — quadratic factors

(d) $y^2 - 81 = (y + 9)(y - 9)$ — difference of squares

(e) $72 - 2t^3 = 2(36 - t^2)$ — common factor then
$= 2(6 + t)(6 - t)$ — difference of squares

(f) $t^4 - 1 = (t^2 + 1)(t^2 - 1)$ — difference of squares
$= (t^2 + 1)(t + 1)(t - 1)$ — repeated

Exercise 4

1 Factorise fully:

(a) $5xy + 15y^2$ **(b)** $7f^2g^2 - fg$

(c) $2pq^2 + 14pq - 7p^2$ **(d)** $rs^3 - 3rs + 6s^2$

(e) $t^2 + 8t + 12$ **(f)** $r^2 - 11r + 10$

(g) $y^2 + 6y + 5$ **(h)** $p^2 - 6p + 8$

(i) $24 - 11s + s^2$ **(j)** $w^2 + 2w - 15$

(k) $v^2 + 3v - 4$ **(l)** $15 + 2w - w^2$

(m) $z^2 + 12z - 13$ **(n)** $-x^2 + 4x - 3$

(o) $25 - 10f + f^2$ **(p)** $a^2 - 6a - 16$

(q) $x^2 - 8x - 9$ **(r)** $35 - 2q - q^2$

(s) $b^2 - b - 20$ **(t)** $h^2 + 14h + 49$

(u) $-1 + 2k - k^2$ **(v)** $3y^2 + 8y + 4$

(w) $2x^2 + 9x + 4$ **(x)** $6m^2 + 13m + 5$

(y) $10d^2 - 11d - 6$ **(z)** $9p^2 + 18p - 16$

2 Factorise fully:

(a) $x^2 - 25$ **(b)** $a^2 - 1$ **(c)** $a^2 - 100$

(d) $4p^2 - 9$ **(e)** $64p^2 - 121$ **(f)** $36 - 25u^2$

(g) $x^2 - 16y^2$ **(h)** $49t^2 - 144s^2$ **(i)** $f^2 - 900g^2$

(j) $5x^2x^2 - 500$ **(k)** $3w^2 - 243$ **(l)** $10v^2 - 40$

(m) $12p^2 - 3$ **(n)** $20 - 45s^2$ **(o)** $7y^2 - 28z^2$

(p) $27a^2 - 48b^2$ **(q)** $125d^2 - 45e^2$ **(r)** $98f^2 - 200g^2$

(s) $v^4 - 16$ **(t)** $w^4 - 81$ **(u)** $x^4 - 10\,000$

(v) $7y^4 - 7$ **(w)** $16a^4 - 1$ **(x)** $162 - 2b^4$

Completing the square

$x^2 + 8x + 16$ is a **perfect square** because $x^2 + 8x + 16 = (x + 4)^2$.

Example 5

Add a number to make $x^2 - 6x$ a perfect square.

Add 9 to make $x^2 - 6x + 9 = (x - 3)^2$

Example 6

Write $x^2 + 8x + 3$ in the form $(x + p)^2 + q$

$x^2 + 8x + 3 = (x^2 + 8x) + 3$ — Separate 3 from the other terms

$= (x^2 + 8x + 16) + 3 - 16$ — Add 16 to complete the square and subtract 16 to maintain value

$= (x + 4)^2 - 13$

Exercise 5

1 Add a number to each expression to make a perfect square.

(a) $x^2 + 2x$ **(b)** $x^2 + 4x$ **(c)** $y^2 + 12y$

(d) $m^2 - 6m$ **(e)** $t^2 - 14t$ **(f)** $w^2 - 20w$

(g) $x^2 + 3x$ **(h)** $a^2 + a$ **(i)** $n^2 + 7n$

(j) $r^2 - 9r$ **(k)** $v^2 - \frac{2}{3}v$ **(l)** $x^2 - \frac{1}{2}x$

2 Write each expression in the form $(x + p)^2 + q$.

(a) $x^2 + 6x + 10$ **(b)** $y^2 - 2y + 3$ **(c)** $z^2 + 8z - 10$

(d) $a^2 - 10a - 5$ **(e)** $b^2 + 18b - 81$ **(f)** $c^2 - 40c + 1$

(g) $r^2 + 5r - 5$ **(h)** $s^2 + s + 2$ **(i)** $t^2 - 3t - 1$

(j) $m^2 + \frac{1}{2}m + \frac{1}{4}$ **(k)** $n^2 + 0.6n - 1$ **(l)** $w^2 - 1.6w + 2$

Algebraic fractions

We can add or subtract algebraic fractions by following the same techniques as we use for numerical fractions.

Example 9

Express each of the following as a single fraction and simplify where possible:

(a) $\frac{4}{x} - \frac{3}{x}$ (b) $\frac{a}{2} + \frac{b}{7}$ (c) $\frac{4}{5y} - \frac{1}{4y}$

(d) $\frac{a}{x} - \frac{b}{y}$ (e) $\frac{x+5}{2} + \frac{x-2}{3}$

(a) $\frac{4}{x} - \frac{3}{x} = \frac{1}{x}$

(b) $\frac{a}{2} + \frac{b}{7} = \frac{a}{2} \times \frac{7}{7} + \frac{b}{7} \times \frac{2}{2}$

$= \frac{7a}{14} + \frac{2b}{14}$

$= \frac{7a + 2b}{14}$

(c) $\frac{4}{5y} - \frac{1}{4y} = \frac{4}{5y} \times \frac{4}{4} - \frac{1}{4y} \times \frac{5}{5}$

$= \frac{16}{20y} - \frac{5}{20y}$

$= \frac{11}{20y}$

(d) $\frac{a}{x} - \frac{b}{y} = \frac{a}{x} \times \frac{y}{y} - \frac{b}{y} \times \frac{x}{x}$

$= \frac{ay}{xy} - \frac{bx}{xy}$

$= \frac{ay - bx}{xy}$

(e) $\frac{x+5}{2} + \frac{x-2}{3} = \frac{(x+5)}{2} \times \frac{3}{3} + \frac{(x-2)}{3} \times \frac{2}{2}$

$= \frac{3(x+5)}{6} + \frac{2(x-2)}{6}$

$= \frac{3x + 15 + 2x - 4}{6}$

$= \frac{5x + 11}{6}$

Exercise 6

1 Express each of the following as a single fraction and simplify where possible:

(a) $\frac{e}{5} + \frac{2e}{5}$ (b) $\frac{5m}{7} - \frac{2m}{7}$ (c) $\frac{5r}{12} + \frac{11r}{12}$

(d) $\frac{a}{3} + \frac{b}{3}$ (e) $\frac{3p}{7} - \frac{q}{7}$ (f) $\frac{13}{v} - \frac{2}{v}$

(g) $\frac{8}{w} - \frac{9}{w}$ (h) $\frac{2a}{u} + \frac{3a}{u}$ (i) $\frac{a}{x} - \frac{b}{x}$

2 Express each of the following as a single fraction and simplify where possible:

(a) $\frac{5x}{6} - \frac{2x}{3}$ (b) $\frac{n}{5} + \frac{3n}{20}$ (c) $\frac{7u}{8} - \frac{5u}{16}$

(d) $\frac{p}{4} - \frac{q}{2}$ (e) $\frac{3x}{25} - \frac{y}{5}$ (f) $\frac{3a}{22} + \frac{3b}{2}$

(g) $\frac{1}{a} + \frac{3}{2a}$ (h) $\frac{5}{3b} - \frac{1}{b}$ (i) $\frac{3}{4g} + \frac{7}{12g}$

(j) $\frac{1}{13p} - \frac{5}{39p}$ (k) $\frac{3}{2x} + \frac{1}{2}$ (l) $\frac{3}{5} - \frac{2}{5a}$

(m) $\frac{a}{2} - \frac{a}{9}$ (n) $\frac{2t}{3} + \frac{t}{8}$ (o) $\frac{3h}{5} - \frac{2h}{7}$

(p) $\frac{u}{3} + \frac{v}{2}$ (q) $\frac{5h}{7} - \frac{4k}{9}$ (r) $\frac{2c}{3} + \frac{3d}{10}$

(s) $\frac{4}{5m} - \frac{1}{2m}$ (t) $\frac{3}{4r} + \frac{5}{6r}$ (u) $\frac{2}{9t} - \frac{3}{4t}$

3 Express each of the following as a single fraction and simplify where possible:

(a) $\frac{a}{2}+\frac{3}{b}$ (b) $\frac{x}{5}-\frac{4}{y}$ (c) $\frac{7}{r}+\frac{s}{9}$

(d) $\frac{8}{u}-\frac{t}{3}$ (e) $\frac{3m}{5}-\frac{7}{2n}$ (f) $\frac{6}{7v}-\frac{2w}{3}$

(g) $\frac{c}{p}+\frac{d}{q}$ (h) $\frac{9}{x}+\frac{5}{y}$ (i) $\frac{8}{b}-\frac{7}{d}$

(j) $\frac{p}{e}-\frac{3}{f}$ (k) $\frac{2a}{7}+\frac{9}{3b}$ (l) $\frac{5}{9}-\frac{x}{2f}$

(m) $\frac{x+1}{2}+\frac{x-2}{3}$ (n) $\frac{a-5}{4}-\frac{a+3}{5}$ (o) $\frac{q+4}{3}+\frac{q-6}{7}$

(p) $\frac{w-7}{8}-\frac{w-1}{9}$ (q) $\frac{t+12}{6}+\frac{t}{10}$ (r) $\frac{2x+3}{5}+\frac{7-x}{9}$

Complex algebraic fractions

We can use the same technique to add or subtract algebraic fractions regardless of how complex they may be.

Example 10

Express as a single fraction:

(a) $\frac{3}{x-2}-\frac{5}{x}$ (b) $\frac{a-5}{a^2+7a+12}+\frac{2}{a+4}$

(a) $\frac{3}{x-2}-\frac{5}{x}=\frac{3}{x-2}\times\frac{x}{x}-\frac{5}{x}\times\frac{(x-2)}{(x-2)}$

$=\frac{3x}{x(x-2)}-\frac{5(x-2)}{x(x-2)}$

$=\frac{3x-5x+10}{x(x-2)}$

$=\frac{-2x+10}{x(x-2)}$

$=\frac{-2(x-5)}{x(x-2)}$

(b) $\dfrac{a-5}{a^2+7a+12}+\dfrac{2}{a+4}=\dfrac{(a-5)}{(a+4)(a+3)}+\dfrac{2}{(a+4)}$

$$=\frac{(a-5)}{(a+4)(a+3)}+\frac{2}{(a+4)}\times\frac{(a+3)}{(a+3)}$$

$$=\frac{(a-5)}{(a+4)(a+3)}+\frac{2(a+3)}{(a+4)(a+3)}$$

$$=\frac{(a-5)+2(a+3)}{(a+4)(a+3)}$$

$$=\frac{a-5+2a+6}{(a+4)(a+3)}$$

$$=\frac{3a+1}{(a+4)(a+3)}$$

Exercise 7

1 Express as a single fraction:

(a) $\dfrac{2}{a+1}+\dfrac{3}{a}$ **(b)** $\dfrac{4}{w+3}-\dfrac{7}{w}$ **(c)** $\dfrac{6}{e-5}+\dfrac{9}{e}$

(d) $\dfrac{5}{m}-\dfrac{8}{m+2}$ **(e)** $\dfrac{1}{r}+\dfrac{9}{r-7}$ **(f)** $\dfrac{7}{v}-\dfrac{3}{v-9}$

(g) $\dfrac{3}{b+1}+\dfrac{2}{b+3}$ **(h)** $\dfrac{4}{n-5}+\dfrac{6}{n+2}$ **(i)** $\dfrac{5}{s+4}-\dfrac{8}{s+7}$

(j) $\dfrac{7}{t-2}-\dfrac{9}{t+5}$ **(k)** $\dfrac{5}{x-1}+\dfrac{1}{x-7}$ **(l)** $\dfrac{9}{y-8}-\dfrac{6}{y-5}$

2 Express as a single fraction:

(a) $\dfrac{2}{a^2-1}+\dfrac{1}{a+1}$ **(b)** $\dfrac{1}{x^2-1}-\dfrac{1}{x-1}$

(c) $\dfrac{1}{b+3}+\dfrac{3}{b^2+4b+3}$ **(d)** $\dfrac{2}{w^2-2w+1}+\dfrac{3}{w-1}$

(e) $\dfrac{1}{p-2}-\dfrac{3}{p^2+p-6}$ **(f)** $\dfrac{2}{x+2}-\dfrac{5}{x^2-3x-10}$

(g) $\dfrac{1}{c^2-2c+1}+\dfrac{1}{c^2-1}$ **(h)** $\dfrac{m+4}{m^2-9}-\dfrac{1}{m-3}$

Indices

There are some rules you need to remember when using indices:

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $a^{-m} = \frac{1}{a^m}$
- $\sqrt[n]{a} = a^{\frac{1}{n}}$
- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
- $a^0 = 1$
- $a^1 = a$

These rules help us to simplify and evaluate expressions containing indices.

Example 11

Simplify:

(a) $3x^2 \times 5x^7$ **(b)** $21a^{\frac{1}{3}} \div 7a^{\frac{1}{4}}$ **(c)** $\left(8b^{-\frac{1}{2}}\right)^{\frac{2}{3}}$

(a) $3x^2 \times 5x^7$
$= 15x^{2+7}$
$= 15x^9$

(b) $21a^{\frac{1}{3}} \div 7a^{\frac{1}{4}}$
$= 3a^{\frac{1}{3}-\frac{1}{4}}$
$= 3a^{\frac{1}{12}}$
$= 3\sqrt[12]{a}$

(c) $\left(8b^{-\frac{1}{2}}\right)^{\frac{2}{3}}$
$= 8^{\frac{2}{3}} b^{-\frac{1}{2}\times\frac{2}{3}}$
$= \left(\sqrt[3]{8}\right)^2 b^{-\frac{1}{3}}$
$= \frac{4}{\sqrt[3]{b}}$

Example 12

Evaluate $243^{-\frac{3}{5}}$.

$$243^{-\frac{3}{5}} = \frac{1}{243^{\frac{3}{5}}} = \frac{1}{\left(\sqrt[5]{243}\right)^3} = \frac{1}{3^3} = \frac{1}{27}$$

Exercise 8

1 Simplify each expression.

(a) $a^5 \times a^4$ **(b)** $n^{-12} \times n^9$ **(c)** $c^6 \times c$ **(d)** $d^{\frac{1}{2}} \times d^3$

(e) $3a^4 \times 5a^3$ **(f)** $4b^9 \times 2b^{-6}$ **(g)** $8c^8 \times 7c$ **(h)** $\frac{v^6}{v^2}$

(i) $y^{19} \div y^{-5}$ **(j)** $\frac{k^8}{k}$ **(k)** $\frac{12c^5}{6c^3}$ **(l)** $\frac{48f^{10}}{6f^{-4}}$

(m) $30c^6 \div c^4$ **(n)** $(c^6)^5$ **(o)** $(y^7)^{-5}$ **(p)** $(6h^5)^3$

(q) $(2x^{-2})^5$ **(r)** $(xy)^5$ **(s)** $(x^2y^3)^4$ **(t)** $(h^3k^5)^{-8}$

2 Evaluate:

(a) $25^{\frac{1}{2}}$ **(b)** $16^{\frac{1}{4}}$ **(c)** $125^{\frac{1}{3}}$ **(d)** $128^{\frac{1}{7}}$ **(e)** $8^{\frac{2}{3}}$

(f) $81^{\frac{3}{4}}$ **(g)** $1000^{\frac{2}{3}}$ **(h)** $243^{\frac{3}{5}}$ **(i)** $625^{-\frac{1}{4}}$ **(j)** $64^{-\frac{5}{6}}$

3 Simplify:

(a) $k^{\frac{1}{2}} \times k^{\frac{1}{4}}$ **(b)** $t^{\frac{2}{3}} \times t^{\frac{2}{3}}$ **(c)** $g^{\frac{3}{4}} \times g^{-\frac{1}{4}}$ **(d)** $\sqrt[3]{y} \times \sqrt[3]{y}$

(e) $4d^{-\frac{1}{2}} \times 5d^{\frac{3}{2}}$ **(f)** $2\sqrt[3]{e} \times 4\sqrt[3]{e^2}$ **(g)** $\dfrac{d^{\frac{2}{7}}}{d^{\frac{1}{5}}}$ **(h)** $d^{\frac{3}{4}} \div d^{\frac{1}{4}}$

(i) $\dfrac{\sqrt[3]{y}}{\sqrt[3]{y}}$ **(j)** $4d^{\frac{1}{2}} \div 5d^{\frac{3}{2}}$ **(k)** $\dfrac{4\sqrt[3]{e}}{2\sqrt[3]{e^2}}$ **(l)** $(4d^{-\frac{1}{2}})^{\frac{3}{2}}$

(m) $(7t)^{-2}$ **(n)** $(c^3d^{\frac{1}{2}})^3$ **(o)** $(x^4y^{\frac{2}{3}})^{\frac{1}{2}}$ **(p)** $(s^{\frac{1}{2}}t^{\frac{2}{5}})^{\frac{2}{3}}$

Complex indices

We can simplify and evaluate complex indices by splitting expressions into separate fractions.

Example 13

Express each fraction as a sum of terms.

(a) $\dfrac{x^4 + x^5}{x^2}$ **(b)** $\dfrac{\sqrt{x} + \sqrt[4]{x^3}}{2\sqrt{x}}$ **(c)** $\dfrac{(\sqrt{x} - 1)^2}{\sqrt{x}}$

(a) $\dfrac{x^4 + x^5}{x^2} = \dfrac{x^4}{x^2} + \dfrac{x^5}{x^2}$

$= x^2 + x^3$

(b) $\dfrac{\sqrt{x} + \sqrt[4]{x^3}}{2\sqrt{x}} = \dfrac{x^{\frac{1}{2}} + x^{\frac{3}{4}}}{2x^{\frac{1}{2}}}$

$= \dfrac{x^{\frac{1}{2}}}{2x^{\frac{1}{2}}} + \dfrac{x^{\frac{2}{4}}}{2x^{\frac{1}{2}}}$

$= \dfrac{x^0}{2} + \dfrac{x^{\frac{1}{4}}}{2}$

$= \dfrac{1}{2} + \dfrac{\sqrt[4]{x}}{2}$

(c) $\dfrac{(\sqrt{x}-1)^2}{\sqrt{x}} = \dfrac{(\sqrt{x}-1)(\sqrt{x}-1)}{\sqrt{x}}$

$$= \frac{x - 2\sqrt{x} + 1}{\sqrt{x}}$$

$$= \frac{x^1}{x^{\frac{1}{2}}} - \frac{2x^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{1}{2}}}$$

$$= x^{\frac{1}{2}} - 2x^0 + \frac{1}{x^{\frac{1}{2}}}$$

$$= \sqrt{x} - 2 + \frac{1}{\sqrt{x}}$$

Exercise 9

1 Express each fraction as a sum or difference of terms.

(a) $\dfrac{x^6 + x^7}{x^3}$ **(b)** $\dfrac{x^{10} - x^{20}}{x^5}$ **(c)** $\dfrac{x^2 - x^3}{2x^2}$

(d) $\dfrac{2x^4 + 3x^2}{x^6}$ **(e)** $\dfrac{x^4 - 1}{x^2}$ **(f)** $\dfrac{x^{\frac{2}{3}} + x^{\frac{1}{3}} + 2}{x^4}$

(g) $\dfrac{x^4 + x^5}{2x^2}$ **(h)** $\dfrac{x^7 + x^2}{3x^4}$ **(i)** $\dfrac{2(x^4 - x^6)}{4x^2}$

(j) $\dfrac{\sqrt[3]{x} + x^2}{x}$ **(k)** $\dfrac{2\sqrt{x} + x^2}{\sqrt{x}}$ **(l)** $\dfrac{6\sqrt[3]{x} + 2\sqrt[4]{x^5}}{\sqrt[3]{x^4}}$

2 Express each fraction as a sum or difference of terms.

(a) $\dfrac{(x+2)^2}{x^3}$ **(b)** $\dfrac{(x+3)(2x-1)}{x}$ **(c)** $\dfrac{(1-x)^2}{2x}$

(d) $\left(\dfrac{3}{x} - 4\right)^2$ **(e)** $\dfrac{(x^2-5)(x+5)}{x^2}$ **(f)** $\left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right)$

(g) $\dfrac{(x-1)^2}{\sqrt{x}}$ **(h)** $\dfrac{(x+2)^2}{x\sqrt{x}}$ **(i)** $\left(\dfrac{1}{\sqrt{x}} + \sqrt{x}\right)^2$

Surds

An **irrational number** is a number that cannot be written as a common fraction. A **surd** is an irrational root, for example:

- $\sqrt{2}$ **is** a surd
- $\sqrt[3]{25}$ **is** a surd
- $\sqrt{\frac{25}{4}}$ is **not** a surd since $\sqrt{\frac{25}{4}} = \frac{5}{2}$
- $\sqrt[3]{125}$ is **not** a surd since $\sqrt[3]{125} = 5$

Example 14

Solve $x^2 + 1 = 4$ leaving your answer in surd form.

$x^2 + 1 = 4$

$x^2 = 3$

$x = \pm\sqrt{3}$

Note: $x = \pm\sqrt{3}$ is an exact answer whereas $x = \pm 1.732\,805\ldots$ is only an approximate answer.

Exercise 10

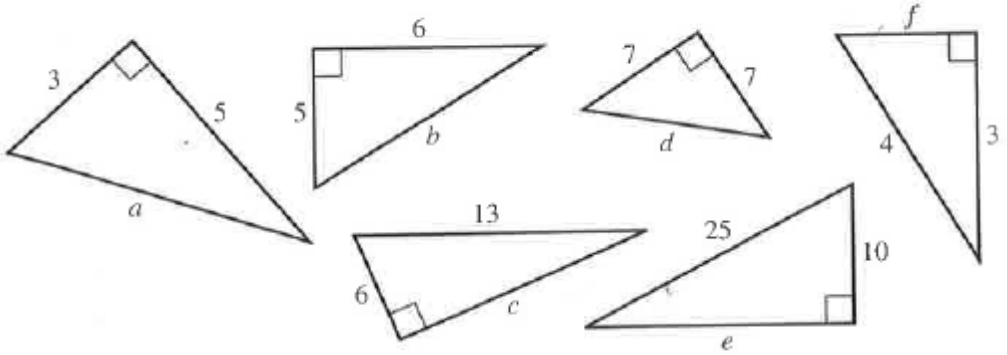
1 Which of these numbers are surds:
$\sqrt{16}, \sqrt{65}, \sqrt[3]{9}, \sqrt[3]{8}, \sqrt{1}, \sqrt[3]{1}, \sqrt{50}, \sqrt[3]{33}, \sqrt[3]{27}, \sqrt{5}, \sqrt{1000}, \sqrt[3]{-1000}$?

2 Find the exact solution of each equation.

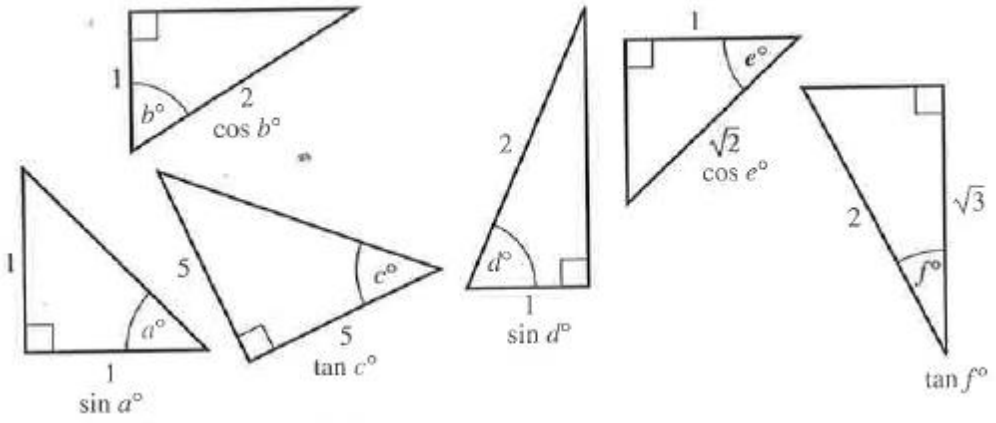
(a) $x^2 - 5 = 9$ **(b)** $x^2 + 6 = 36$ **(c)** $x^3 - 4 = 60$

(d) $x^2 + 11 = 12$ **(e)** $x^3 - 13 = 26$ **(f)** $x^3 + 20 = 19$

3 For each triangle find the **exact** length of the unknown side.



4 Find the **exact** value of each trigonometry ratio.



Simplifying surds

A surd can be simplified if it has a factor that is a perfect square, because of the rule:

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

Surds can be simplified using the normal rules of algebra.

Example 15

Express $\sqrt{18}$ in simplest form.

Find the largest square number that is a factor

$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$

Example 16

Simplify $3\sqrt{2} - 5\sqrt{2} + 4\sqrt{5}$.

$3\sqrt{2} - 5\sqrt{2} + 4\sqrt{5} = -2\sqrt{2} + 4\sqrt{5}$

$= 4\sqrt{5} - 2\sqrt{2}$

Exercise 11

1 Express in simplest form:

(a) $\sqrt{12}$ **(b)** $\sqrt{20}$ **(c)** $\sqrt{27}$ **(d)** $\sqrt{32}$

(e) $\sqrt{45}$ **(f)** $\sqrt{48}$ **(g)** $\sqrt{50}$ **(h)** $\sqrt{63}$

(i) $\sqrt{75}$ **(j)** $\sqrt{44}$ **(k)** $\sqrt{98}$ **(l)** $\sqrt{500}$

(m) $5\sqrt{8}$ **(n)** $3\sqrt{18}$ **(o)** $4\sqrt{200}$ **(p)** $3\sqrt{1000}$

2 Simplify:

(a) $7\sqrt{2} + 3\sqrt{2}$ **(b)** $9\sqrt{5} - 5\sqrt{5}$ **(c)** $\sqrt{3} + 6\sqrt{3}$

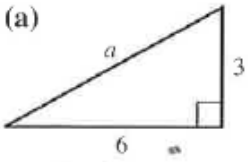
(d) $4\sqrt{7} - \sqrt{7}$ **(e)** $9\sqrt{10} - 9\sqrt{10}$ **(f)** $\sqrt{5} - 8\sqrt{5}$

(g) $3\sqrt{2} - \sqrt{2} + 7\sqrt{2}$ **(h)** $\sqrt{7} + \sqrt{5} + 2\sqrt{7}$ **(i)** $2\sqrt{10} - 10\sqrt{2}$

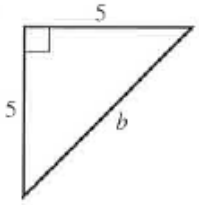
(j) $2\sqrt{5} + 3\sqrt{2} - 2\sqrt{5} - \sqrt{2}$ **(k)** $-4\sqrt{11} + 8\sqrt{10} - 2\sqrt{11} - 2\sqrt{10}$

3 Calculate the **exact** length of the unknown side in each triangle. Write each answer in its simplest form.

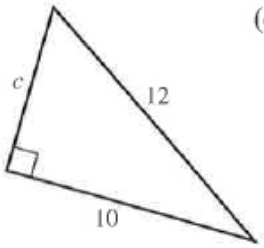
(a)



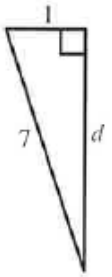
(b)



(c)



(d)



4 Solve these equations, where necessary leaving the answer as a surd in its simplest form.

(a) $x^2 + 8 = 36$ **(b)** $x^2 - 15 = 60$ **(c)** $\frac{1}{2}x^2 + 2 = 51$

(d) $x^2 - 147 = 0$ **(e)** $x^3 + 12 = 4$ **(f)** $x^3 - 5 = 49$

Multiplication of surds

Surds can be multiplied using the following rules:

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\sqrt{a} \times \sqrt{a} = a$

Example 17

Simplify:

(a) $\sqrt{8} \times \sqrt{10}$ **(b)** $(3 + \sqrt{2})(3 - \sqrt{2})$ **(c)** $(\sqrt{3} + \sqrt{2})^2$

(a) $\sqrt{8} \times \sqrt{10} = \sqrt{8 \times 10}$
$= \sqrt{80}$
$= \sqrt{16} \times \sqrt{5}$
$= 4\sqrt{5}$

or

$R8 \times \sqrt{10} = \sqrt{4} \times 2 \times \sqrt{2} \times 5$
$= 2 \times \sqrt{2} \times \sqrt{2} \times \sqrt{5}$
$= 2 \times 2 \times \sqrt{5}$
$= 4\sqrt{5}$

(b) $(3 + \sqrt{2})(3 - \sqrt{2}) = 3 \times 3 - 3 \times \sqrt{2} + \sqrt{2} \times 3 - \sqrt{2} \times \sqrt{2}$
$= 9 - 3\sqrt{2} + 3\sqrt{2} - 2$
$= 7$

(c) $(\sqrt{3} + \sqrt{2})^2 = (\sqrt{3} + \sqrt{2})(\sqrt{3} + \sqrt{2})$
$= \sqrt{3} \times \sqrt{3} + \sqrt{3} \times \sqrt{2} + \sqrt{2} \times \sqrt{3} + \sqrt{2} \times \sqrt{2}$
$= 3 + \sqrt{6} + \sqrt{6} + 2$
$= 5 + 2\sqrt{6}$

Exercise 12

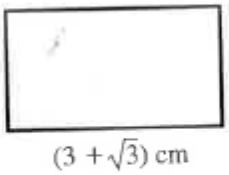
1 Simplify:

(a) $\sqrt{3} \times \sqrt{3}$ **(b)** $\sqrt{7} \times \sqrt{7}$ **(c)** $\sqrt{2a} \times \sqrt{2a}$
(d) $\sqrt{4} \times \sqrt{3}$ **(e)** $\sqrt{9} \times \sqrt{2}$ **(f)** $\sqrt{3} \times \sqrt{25}$
(g) $\sqrt{2} \times \sqrt{5}$ **(h)** $\sqrt{7} \times \sqrt{3}$ **(i)** $\sqrt{11} \times \sqrt{2}$
(j) $\sqrt{2} \times \sqrt{8}$ **(k)** $\sqrt{12} \times \sqrt{3}$ **(l)** $\sqrt{2} \times \sqrt{50}$
(m) $\sqrt{2} \times \sqrt{10}$ **(n)** $\sqrt{3} \times \sqrt{6}$ **(o)** $\sqrt{8} \times \sqrt{12}$
(p) $\sqrt{10} \times \sqrt{20}$ **(q)** $3\sqrt{2} \times 5\sqrt{2}$ **(r)** $3\sqrt{5} \times 5\sqrt{3}$

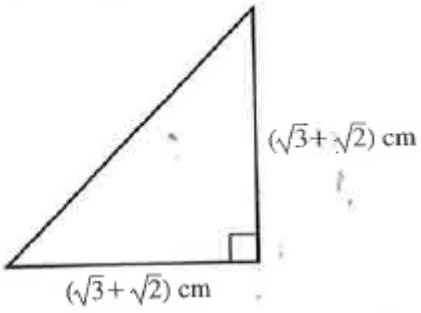
2 Simplify:

(a) $\sqrt{2}(1 + \sqrt{2})$ **(b)** $\sqrt{3}(\sqrt{3} - 1)$
(c) $(1 + \sqrt{5})\sqrt{5}$ **(d)** $\sqrt{7}(5 + \sqrt{7})$
(e) $\sqrt{2}(3 - 2\sqrt{2})$ **(f)** $(3\sqrt{5} - 2)\sqrt{5}$
(g) $(\sqrt{3} + 1)(\sqrt{3} - 1)$ **(h)** $(\sqrt{5} - 2)(\sqrt{5} + 2)$
(i) $(3 + \sqrt{7})(3 - \sqrt{7})$ **(j)** $(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})$
(k) $(\sqrt{7} - \sqrt{13})(\sqrt{7} + \sqrt{13})$ **(l)** $(2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2})$
(m) $(1 + \sqrt{3})^2$ **(n)** $(\sqrt{5} - 2)^2$
(o) $(\sqrt{2} + \sqrt{7})^2$ **(p)** $(\sqrt{3} - \sqrt{5})^2$

3 (a) Calculate the **exact** area of the rectangle.



(b) Calculate the **exact** length of the hypotenuse.



Rationalising denominators

It is sometimes convenient to work with fractions that do not have a surd in the denominator. We can obtain fractions with **rational denominators** by multiplying by $\frac{\sqrt{x}}{\sqrt{x}}$ $(= 1)$.

Example 18

Express $\frac{5}{\sqrt{3}}$ with a rational denominator.

$$\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

Exercise 13

1 Rationalise the denominators of these fractions:

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{5}}$ **(c)** $\frac{6}{\sqrt{3}}$ **(d)** $\frac{8}{\sqrt{2}}$ **(e)** $\frac{2}{\sqrt{3}}$

(f) $\frac{10}{\sqrt{5}}$ **(g)** $\frac{7}{\sqrt{3}}$ **(h)** $\frac{3}{\sqrt{5}}$ **(i)** $\frac{4}{5\sqrt{2}}$ **(j)** $\frac{7}{2\sqrt{5}}$

2 Rationalise the denominator of these fractions then simplify:

(a) $\frac{1}{\sqrt{20}}$ **(b)** $\frac{1}{\sqrt{50}}$ **(c)** $\frac{10}{\sqrt{12}}$ **(d)** $\frac{7}{\sqrt{18}}$ **(e)** $\frac{2}{\sqrt{75}}$

3 Write these fractions in their simplest form with a rational denominator:

(a) $\frac{\sqrt{9}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{5}}{\sqrt{3}}$ **(c)** $\sqrt{\frac{9}{10}}$ **(d)** $\sqrt{\frac{1}{3}}$ **(e)** $\sqrt{\frac{3}{5}}$

Exercise 14 - Solving Trigonometric Equations

Google: mrgrahammaths National 5 trigonometric equations for video help

1. Find the two solutions for each of the following in the range $0 \leq x \leq 360$:
(Give each answer correct to the nearest whole degree).

(a) $\sin x° = 0{\cdot}500$	(b) $\cos x° = 0{\cdot}707$	(c) $\tan x° = 0{\cdot}869$
(d) $\cos x° = 0{\cdot}940$	(e) $\tan x° = 1{\cdot}280$	(f) $\sin x° = 0{\cdot}574$
(g) $\sin x° = 0{\cdot}990$	(h) $\tan x° = 6{\cdot}314$	(i) $\cos x° = 0{\cdot}391$
(j) $\cos x° = 0{\cdot}985$	(k) $\sin x° = 0{\cdot}866$	(l) $\tan x° = 1{\cdot}732$

2. Rearrange each of the following and solve them in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).

(a) $2\cos x° - 1 = 0$	(b) $5\sin x° - 4 = 0$	(c) $10\tan x° - 7 = 0$
(d) $1 - 3\sin x° = 0$	(e) $5 - 6\cos x° = 0$	(f) $3\tan x° - 5 = 0$

3. Find the two solutions for each of the following in the range $0 \leq x \leq 360$:
(Give each answer correct to the nearest whole degree).

(a) $\sin x° = -0{\cdot}500$	(b) $\cos x° = -0{\cdot}707$	(c) $\tan x° = -0{\cdot}384$
(d) $\cos x° = -0{\cdot}292$	(e) $\tan x° = -1{\cdot}000$	(f) $\sin x° = -0{\cdot}866$
(g) $\tan x° = -4$	(h) $\sin x° = -0{\cdot}174$	(i) $\cos x° = -0{\cdot}927$

4. Rearrange each of the following and solve them in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).

(a) $4\sin x° + 1 = 0$	(b) $5\cos x° + 3 = 0$	(c) $3\tan x° + 1 = 0$
(d) $7 + 8\cos x° = 0$	(e) $0{\cdot}4\sin x° + 0{\cdot}3 = 0$	(f) $5\tan x° + 8 = 0$

5. Solve the following mixture of trigonometric equations in the range $0 \leq x \leq 360$.
(Give your answers correct to 1 decimal place).

(a) $\sin x° = 0{\cdot}323$	(b) $\cos x° = -0{\cdot}9$	(c) $\tan x° = 0{\cdot}678$
(d) $\cos x° = {}^1/_4$	(e) $\sin x° = -0{\cdot}707$	(f) $\tan x° = -2$
(g) $\sin x° = {}^3/_5$	(h) $\cos x° = -0{\cdot}111$	(i) $\tan x° = {}^5/_8$
(j) $8\sin x° + 5 = 0$	(k) $6\cos x° + 3 = 0$	(l) $1 - 5\tan x° = 0$
(m) $20\sin x° - 17 = 0$	(n) $15 - 25\cos x° = 0$	(o) $8\tan x° + 7 = 0$
(p) $5\sin x° + 3 = 2\sin x° + 5$	(q) $7\cos x° - 1 = \cos x° + 4$	
(r) $10\tan x° + 8 = 3\tan x° + 4$	(s) $6\sin x° + 11 = 3\sin x° + 10$	

Revision exercise 3C

1 (a) K (1, 2, 0), L (1, 4.5, −0.5), M (2, 1.5, 0.5), N (2, −1, 1)
(b) Parallelogram. The opposite sides are parallel.

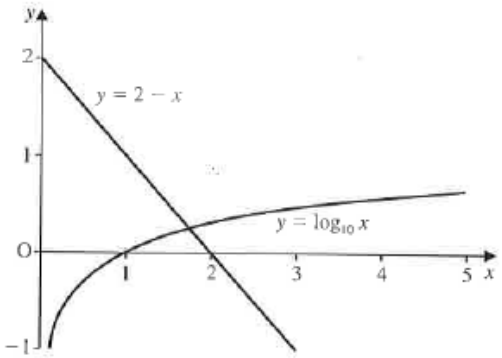
2 (a) 30°, 150° (b) $\sqrt{3} + \frac{\sqrt{3}}{2}$

3 (a) T (3, 4, 1) (c) 51.3°

4 (a) 88.69 mg (b) 2972 years (c) 5776 years

5 (a) $10 \sin(4t + 53.1)°$ (b) 10
(c) Max: 0.051π, 0.55π; min: 0.30π, 0.80π

6 (a)



$x \approx 2$

(b) 1.76

7 (a) $k = 0.067$ (b) Yes. $P_4 > 30$

8 (a) A (1, 0, 0) B (3, 2, 0) C (3, 0, −2)
(b) $2\sqrt{3}$ unit² (c) 4.7% decrease

9 (a) $\frac{3\pi - 4}{6}, \frac{5\pi - 4}{6}, \frac{7\pi - 4}{6}, \frac{9\pi - 4}{6}, \frac{11\pi - 4}{6}, \frac{13\pi - 4}{6}$
(b) −11.87 m/s² (c) 2.49 m

10 (a) 10 m (b) $36\frac{1}{4}$ m² (c) 2900 m³

11 (a) $600 \cos\left(\pi t + \frac{7\pi}{6}\right)$
(b) Period = 2, amplitude = 600 (c) After 0.2 sec

12 (a) -9.9×10^{-10} (b) 2.56×10^7 years
(c) 0.55%

13 (a) $4 \cos\theta + 10 \sin\theta$ (b) $\sqrt{116} \sin(\theta + 0.38)$
(c) $10 \cos\theta - 4 \sin\theta$ (d) $\sqrt{116}$

14 (c) $x = 4, T = 10$

15 (b) 0.0132

Essential skills

Exercise 1

1 (a) $8pq$ (b) $7xy + 2zy$
(c) $7km$ (d) $7cd^2$
(e) $13rs^2 - 8r^2s$ (f) $7ax^2 + 3bx^2$
(g) $7v^2w$ (h) $8p^2q^2$
(i) $y^2 + 6y$ (j) $3r^3 - 3r$
(k) $4b^2 - 3ab$ (l) $6x^2$
(m) $6ab + 3xy$ (n) $8pq - kl$
(o) $2xy^2 + xy$ (p) $8vw^2 - 7v^3w$
(q) $-6cd$ (r) $a^3 + b^3 - a^3b^3$
(s) $x^3 + 3x^2$ (t) $x^4 - 4x^2$

2 (a) 20 (b) $2p^2 + 3p$
(c) $-m - 2$ (d) $-2b^2 + 8$
(e) $2a + b + 3c$ (f) $6p - 5q - 2r$
(g) $3x^2 - 7xy$ (h) $s^2 - 8r^2s^2 - 5r^2$
(i) $6k - 0.4j$ (j) $\frac{3}{4}t^2 - \frac{3}{2}s + \frac{3}{4}$
(k) $4x^2 + 2xy - 10y^2$ (l) $6a^2b - 8ab + b^2a$

Exercise 2

1 (a) $21a^3b$ (b) $-32x^2y^2$ (c) m^3n^3
(d) $abpq$ (e) k^2m^2 (f) $-cd^3$
(g) 1 (h) pr (i) $\frac{1}{w}$
(j) $12b^3$ (k) $125ef^2$ (l) $7fg^3$
(m) $49x^4$ (n) $-4p^3q^2g^2$ (o) $\frac{1}{a}$
(p) $\frac{6r}{s}$ (q) y (r) $\frac{1}{ab}$

2 (a) $2b$ (b) $3x$ (c) 4
(d) $7q$ (e) $5r$ (f) $6fg$
(g) $3\frac{b}{a}$ (h) $\frac{2}{v}$ (i) $\frac{19}{b}$
(j) $\frac{m}{6}$ (k) $\frac{g}{2h}$ (l) $4l$
(m) $\frac{1}{13tu}$ (n) $3w^2$ (o) $\frac{7y}{x}$

3 (a) $15x^2 + 6x$ (b) $7y^3 - 7yz$ (c) $-6a^2 + 15a^4$
(d) $-9p + q$ (e) $12rs^2 + 12r^2$ (f) $-vw^3 - w^2$
(g) $39xy - 15y^2$ (h) $a^3bc + ab^3c$ (i) $-5d^5e - e^3$
(j) $3f^2g^2 - 2f^3g$ (k) $km^3 + 5kmn$ (l) $-8t^3u + 8t^4u$

4 (a) $9x + 6$ (b) $7x^2 - y$ (c) $x - 5$
(d) $3b + \frac{a}{4}$ (e) $6pq - 2q$ (f) $5rs + 7s$
(g) $1 + pr$ (h) $2x^2y - 4 + x$

Exercise 3

1 (a) $x^2 + 7x + 12$ (b) $24 - 10y + y^2$
(c) $p^2 - 15p + 26$ (d) $r^2 - r - 30$
(e) $32 - 4s - s^2$ (f) $2t^2 - 9t - 35$
(g) $21v^2 + 5v - 4$ (h) $8 - 42w + 27w^2$
(i) $2x^2 + 24x + 70$ (j) $5a^2 - 35a - 90$
(k) $28 - 35t + 7t^2$ (l) $-4m^2 + 32m + 80$

2 (a) $x^2 + 4x + 4$ (b) $9y^2 - 30y + 25$
(c) $49 + 28a + 4a^2$ (d) $81 - 72q + 16q^2$
(e) $x^2 - 4$ (f) $9k^2 - 1$
(g) $400 - 49q^2$ (h) $3x^2 - 3$
(i) $5a^2 - 20$ (j) $90 - 10p^2$
(k) $6m^2 - 6n^2$ (l) $18k^2 - 32j^2$
(m) $b^2 + 10b + 25$ (n) $d^2 - 12d + 36$
(o) $64 - 16k + k^2$ (p) $100 + 20p + p^2$
(q) $81 + 36z + 4z^2$ (r) $49 - 70v + 25v^2$
(s) $9p^2 - 6pq + q^2$ (t) $25m^2 + 20mn + 4n^2$
(u) $a^2 + 2ab + b^2$ (v) $g^2 - 14gh + 49h^2$
(w) $25x^2 - 80xy + 64y^2$ (x) $36v^2 + 108vw + 81w^2$

3 (a) $-3x^2 + 7x + 30$ (b) $5x^3 + 4x^2 + 8x$
(c) $37x^2 - 72x - 48$ (d) $y^3 + 4y^2 + 3y - 25$
(e) $-4a^3 - 3a^2 - 117a + 324$

(f) $24p^2 + 20p - 25$ (g) $4f^3 - 58f^2 + 132f - 2$
(h) $9t^3 - 18t - 9$ (i) $36 - 318s - 80s^2 - 6s^3$
(j) $3w^2 - 14w - 12$

4 (a) $x^3 - x^2 + x + 3$ (b) $2y^3 + 14y^2 + 8y - 24$
(c) $15a^3 - 41a^2 + 34a - 8$ (d) $6b^3 + 29b^2 - 17b + 2$
(e) $21p^3 + 18p^2 - 75p - 54$ (f) $5 - 7q + 3q^2 - q^3$
(g) $64 - 16t + 21t^2 + 3t^3$ (h) $54 - 66s + 2s^2 + 10s^3$
(i) $3x^3 + 11x^2 + 11x + 2$ (j) $-7d^3 + 40d^2 + 20d - 48$
(k) $45 - 58f + 29f^2 + 4f^3$ (l) $18 - 24h - 51h^2 - 18h^3$
(m) $2w^3 + 4w^2 - 28w + 10$ (n) $6z^3 + 39z^2 - 33z + 6$
(o) $-15a^3 + 25a^2 + 20a - 20$
(p) $28 + 20b - 12b^2 - 4b^3$

5 (a) $k^3 + 12k^2 + 48k + 64$
(b) $27r^3 - 135r^2 + 225r - 125$
(c) $p^3 - 3p^2 + 3p - 1$ (d) $64 + 96q + 48q^2 + 8q^3$
(e) $a^3 + 6a^2 + 12a + 8$ (f) $b^3 - 3b^2 + 3b - 1$
(g) $8d^3 + 36d^2 + 54d + 27$ (h) $125e^3 - 150e^2 + 60e - 8$
(i) $1 + 12x + 48x^2 + 64x^3$

Exercise 4

1 (a) $5y(x + 3y)$ (b) $fg(7fg - 1)$
(c) $p(2q^2 + 14q - 7p)$ (d) $s(rs^2 - 3r + 6s)$
(e) $(t + 6)(t + 2)$ (f) $(r - 10)(r - 1)$
(g) $(y + 5)(y + 1)$ (h) $(p - 4)(p - 2)$
(i) $(8 - s)(3 - s)$ (j) $(w + 5)(w - 3)$
(k) $(v + 4)(v - 1)$ (l) $(5 - w)(3 + w)$
(m) $(z + 13)(z - 1)$ (n) $(1 - x)(x - 3)$
(o) $(5 - f)^2$ (p) $(a + 2)(a - 8)$
(q) $(x - 9)(x + 1)$ (r) $(7 + q)(5 - q)$
(s) $(b - 5)(b + 4)$ (t) $(h + 7)^2$
(u) $-(k - 1)^2$ (v) $(3y + 2)(y + 2)$
(w) $(2x + 1)(x + 4)$ (x) $(3m + 5)(2m + 1)$
(y) $(5d + 2)(2d - 3)$ (z) $(3p + 8)(3p - 2)$

2 (a) $(x + 5)(x - 5)$ (b) $(a + 1)(a - 1)$
(c) $(a + 10)(a - 10)$ (d) $(2p + 3)(2p - 3)$
(e) $(8p + 11)(8p - 11)$ (f) $(6 + 5u)(6 - 5u)$
(g) $(x + 4y)(x - 4y)$ (h) $(7t + 12s)(7t - 12s)$
(i) $(f + 30g)(f - 30g)$ (j) $5(x + 10)(x - 10)$
(k) $3(w + 9)(w - 9)$ (l) $10(v - 2)(v + 2)$
(m) $3(2p + 1)(2p - 1)$ (n) $5(2 + 3s)(2 - 3s)$
(o) $7(y + 2z)(y - 2z)$ (p) $3(3a + 4b)(3a - 4b)$
(q) $5(5d + 3e)(5d - 3e)$ (r) $2(7f + 10g)(7f - 10g)$
(s) $(v^2 + 4)(v + 2)(v - 2)$ (t) $(w^2 + 9)(w + 3)(w - 3)$
(u) $(x^2 + 100)(x + 10)(x - 10)$
(v) $7(y^2 + 1)(y + 1)(y - 1)$
(w) $(4a^2 + 1)(2a + 1)(2a - 1)$
(x) $2(9 + b^2)(3 + b)(3 - b)$

Exercise 5

1 (a) $x^2 + 2x + 1$ (b) $x^2 + 4x + 4$
(c) $y^2 + 12y + 36$ (d) $m^2 - 6m + 9$
(e) $t^2 - 14t + 49$ (f) $w^2 - 20w + 100$
(g) $x^2 + 3x + \frac{9}{4}$ (h) $a^2 + a + \frac{1}{4}$
(i) $n^2 + 7n + \frac{49}{4}$ (j) $r^2 - 9r + \frac{81}{4}$
(k) $v^2 - \frac{2}{3}v + \frac{1}{9}$ (l) $x^2 - \frac{1}{2}x + \frac{1}{16}$

2 (a) $(x + 3)^2 + 1$ (b) $(y - 1)^2 + 2$
(c) $(z + 4)^2 - 26$ (d) $(a - 5)^2 - 30$
(e) $(b + 9)^2 - 162$ (f) $(c - 20)^2 - 399$
(g) $(r + \frac{5}{2})^2 - \frac{45}{4}$ (h) $(s + \frac{1}{2})^2 + \frac{7}{4}$
(i) $(t - \frac{3}{2})^2 - \frac{13}{4}$ (j) $(m + \frac{1}{4})^2 + \frac{3}{16}$
(k) $(n + 0.3)^2 - 1.09$ (l) $(w - 0.8)^2 + 1.36$

3 (a) $5 - (x - 1)^2$ (b) $9 - (x + 2)^2$
(c) $\frac{9}{4} - (x - \frac{1}{2})^2$ (d) $\frac{33}{4} - (x - \frac{3}{2})^2$
(e) $-5 - (x - 1)^2$ (f) $-2 - (x - 2)^2$

4 (a) $2(x + 1)^2 - 3$ (b) $3(y + 2)^2 - 7$
(c) $5(a - 3)^2 - 63$ (d) $2(n + \frac{1}{2})^2 + \frac{1}{2}$
(e) $4(b - \frac{3}{2})^2 - 14$ (f) $2(m - \frac{3}{4})^2 - \frac{57}{8}$

Exercise 6

1 (a) $\frac{3e}{5}$ (b) $\frac{3m}{7}$ (c) $\frac{4r}{3}$
(d) $\frac{a + b}{3}$ (e) $\frac{3p - q}{7}$ (f) $\frac{11}{v}$
(g) $\frac{-1}{w}$ (h) $\frac{5a}{u}$ (i) $\frac{a - b}{x}$

2 (a) $\frac{x}{6}$ (b) $\frac{7n}{20}$ (c) $\frac{9u}{16}$
(d) $\frac{p - 2q}{4}$ (e) $\frac{3x - 5y}{25}$ (f) $\frac{3a + 33b}{22}$
(g) $\frac{5}{2a}$ (h) $\frac{2}{3b}$ (i) $\frac{4}{3g}$
(j) $\frac{-2}{39p}$ (k) $\frac{3 + x}{2x}$ (l) $\frac{3a - 2}{5a}$
(m) $\frac{7a}{18}$ (n) $\frac{19t}{24}$ (o) $\frac{11h}{35}$
(p) $\frac{2u + 3v}{6}$ (q) $\frac{45h - 28k}{63}$ (r) $\frac{20c + 9d}{30}$
(s) $\frac{3}{10m}$ (t) $\frac{19}{12r}$ (u) $\frac{-19}{36t}$

3 (a) $\frac{ab + 6}{2b}$ (b) $\frac{xy - 20}{5y}$ (c) $\frac{63 + rs}{9r}$
(d) $\frac{24 - tu}{3u}$ (e) $\frac{6mn - 35}{10n}$ (f) $\frac{18 - 14vw}{21v}$
(g) $\frac{cq + dp}{pq}$ (h) $\frac{9y + 5x}{xy}$ (i) $\frac{8d - 7b}{bd}$
(j) $\frac{fp - 3e}{ef}$ (k) $\frac{6ab + 63}{21b}$ (l) $\frac{10f - 9x}{18f}$
(m) $\frac{5x - 1}{6}$ (n) $\frac{a - 37}{20}$ (o) $\frac{10q + 10}{21}$
(p) $\frac{w - 55}{72}$ (q) $\frac{4t + 30}{15}$ (r) $\frac{13x + 62}{45}$

Exercise 7

1 (a) $\frac{5a + 3}{a(a + 1)}$ (b) $\frac{-3w - 21}{w(w + 3)}$
(c) $\frac{15e - 45}{e(e - 5)}$ (d) $\frac{10 - 3m}{m(m + 2)}$

(e) $\frac{10r-7}{r(r-7)}$ (f) $\frac{4v-63}{v(v-9)}$

(g) $\frac{5b+11}{(b+1)(b+3)}$ (h) $\frac{10n-22}{(n-5)(n+2)}$

(i) $\frac{3-3s}{(s+4)(s+7)}$ (j) $\frac{53-2t}{(t-2)(t+5)}$

(k) $\frac{6x-36}{(x-1)(x-7)}$ (l) $\frac{3y+3}{(y-8)(y-5)}$

2 (a) $\frac{1}{(a-1)}$ (b) $\frac{-x}{(x+1)(x-1)}$

(c) $\frac{b+4}{(b+3)(b+1)}$ (d) $\frac{3w-1}{(w-1)^2}$

(e) $\frac{p}{(p-2)(p+3)}$ (f) $\frac{2x-15}{(x-5)(x+2)}$

(g) $\frac{2c}{(c+1)(c-1)(c-1)}$ (h) $\frac{1}{(m+3)(m-3)}$

Exercise 8

1 (a) a^9 (b) n^{-3} (c) c^7 (d) $d^{\frac{1}{2}}$

(e) $15a^7$ (f) $8b^3$ (g) $56c^9$ (h) v^4

(i) y^{24} (j) k^7 (k) $2c^2$ (l) $8f^{14}$

(m) $30c^2$ (n) c^{30} (o) y^{-35} (p) $216h^{15}$

(q) $32x^{-16}$ (r) x^5y^5 (s) x^8y^{12} (t) $h^{-24}k^{-40}$

2 (a) 5 (b) 2 (c) 5 (d) 2

(e) 4 (f) 27 (g) 100 (h) 27

(i) $\frac{1}{5}$ (j) $\frac{1}{32}$

3 (a) $k^{\frac{1}{2}}$ (b) $t^{\frac{1}{3}}$ (c) $g^{\frac{1}{3}}$ (d) $y^{\frac{2}{3}}$

(e) $20d$ (f) $8e$ (g) $d^{\frac{1}{2}}$ (h) $d^{\frac{1}{3}}$

(i) 1 (j) $\frac{4}{5}d^{-1}$ (k) $2e^{-\frac{1}{2}}$ (l) $8d^{-\frac{2}{3}}$

(m) $\frac{1}{49t^2}$ (n) $c^9d^{\frac{1}{2}}$ (o) x^2y (p) $s^{\frac{1}{3}}t^{\frac{4}{5}}$

Exercise 9

1 (a) $x^3 + x^4$ (b) $x^5 - x^{15}$

(c) $\frac{1}{2} - \frac{1}{2}x$ (d) $2x^{-2} + 3x^{-4}$

(e) $x^2 - x^{-2}$ (f) $x^{-\frac{16}{3}} + x^{-\frac{11}{2}} + 2x^{-4}$

(g) $\frac{1}{2}x^2 + \frac{1}{2}x^3$ (h) $\frac{1}{3}x^3 + \frac{1}{3}x^{-2}$

(i) $\frac{1}{2}x^2 - \frac{1}{2}x^6$ (j) $x^{-\frac{1}{2}} + x$

(k) $2 + x^{\frac{1}{2}}$ (l) $6x^{-1} + 2x^{-\frac{1}{2}}$

2 (a) $x^{-1} + 4x^{-2} + 4x^{-3}$ (b) $2x + 5 - 3x^{-1}$

(c) $\frac{1}{2}x^{-1} - 1 + \frac{1}{2}x$ (d) $9x^{-2} - 24x^{-1} + 16$

(e) $x + 5 - 5x^{-1} - 25x^{-2}$ (f) $x^2 - \frac{1}{x^2}$

(g) $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} + x^{-\frac{1}{2}}$ (h) $x^{\frac{1}{2}} + 4x^{-\frac{1}{2}} + 4x^{-\frac{3}{2}}$

(i) $\frac{1}{x} + 2 + x$

Exercise 10

1 $\sqrt{65}, \sqrt[3]{9}, \sqrt{50}, \sqrt[3]{33}, \sqrt{5}, \sqrt{1000}$,

2 (a) $x = \pm\sqrt{14}$ (b) $x = \pm\sqrt{30}$ (c) $x = 4$

(d) $x = \pm 1$ (e) $x = \sqrt[3]{39}$ (f) $x = -1$

3 $a = \sqrt{34}, b = \sqrt{61}, c = \sqrt{133}, d = \sqrt{98}, e = \sqrt{525}, f = \sqrt{7}$

4 $\sin a^\circ = \frac{1}{\sqrt{2}}, \cos b^\circ = \frac{1}{2}, \tan c^\circ = 1, \sin d^\circ = \frac{\sqrt{3}}{2}$,

$\cos e^\circ = \frac{1}{\sqrt{2}}, \tan f^\circ = \frac{1}{\sqrt{3}}$

Exercise 11

1 (a) $2\sqrt{3}$ (b) $2\sqrt{5}$ (c) $3\sqrt{3}$ (d) $4\sqrt{2}$

(e) $3\sqrt{5}$ (f) $4\sqrt{3}$ (g) $5\sqrt{2}$ (h) $3\sqrt{7}$

(i) $5\sqrt{3}$ (j) $2\sqrt{11}$ (k) $7\sqrt{2}$ (l) $10\sqrt{5}$

(m) $10\sqrt{2}$ (n) $9\sqrt{2}$ (o) $40\sqrt{2}$ (p) $30\sqrt{10}$

2 (a) $10\sqrt{2}$ (b) $4\sqrt{5}$ (c) $7\sqrt{3}$ (d) $3\sqrt{7}$

(e) 0 (f) $-7\sqrt{5}$ (g) $9\sqrt{2}$ (h) $3\sqrt{7} + \sqrt{5}$

(i) $2\sqrt{10} - 10\sqrt{2}$ (j) $2\sqrt{2}$

(k) $6\sqrt{10} - 6\sqrt{11}$

3 (a) $3\sqrt{5}$ (b) $5\sqrt{2}$ (c) $2\sqrt{11}$ (d) $4\sqrt{3}$

4 (a) $\pm 2\sqrt{7}$ (b) $\pm 5\sqrt{3}$ (c) $\pm 7\sqrt{2}$ (d) $\pm 7\sqrt{3}$

(e) -2 (f) $3\sqrt[3]{2}$

Exercise 12

1 (a) 3 (b) 7 (c) $2a$

(d) $2\sqrt{3}$ (e) $3\sqrt{2}$ (f) $5\sqrt{3}$

(g) $\sqrt{10}$ (h) $\sqrt{21}$ (i) $\sqrt{22}$

(j) 4 (k) 6 (l) 10

(m) $2\sqrt{5}$ (n) $3\sqrt{2}$ (o) $4\sqrt{6}$

(p) $10\sqrt{2}$ (q) 30 (r) $15\sqrt{15}$

2 (a) $\sqrt{2} + 2$ (b) $3 - \sqrt{3}$ (c) $\sqrt{5} + 5$

(d) $5\sqrt{7} + 7$ (e) $3\sqrt{2} - 4$ (f) $15 - 2\sqrt{5}$

(g) 2 (h) 1 (i) 2

(j) 3 (k) -6 (l) -6

(m) $4 + 2\sqrt{3}$ (n) $9 - 4\sqrt{5}$ (o) $9 + 2\sqrt{14}$

(p) $8 - 2\sqrt{15}$

3 (a) $6\,\text{cm}^2$ (b) $\sqrt{(10 + 4\sqrt{6})}\,\text{cm}$

Exercise 13

1 (a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{5}}{5}$ (c) $2\sqrt{3}$

(d) $4\sqrt{2}$ (e) $\frac{2\sqrt{3}}{3}$ (f) $2\sqrt{5}$

(g) $\frac{7\sqrt{3}}{3}$ (h) $\frac{3\sqrt{5}}{5}$ (i) $\frac{2\sqrt{2}}{5}$

(j) $\frac{7\sqrt{5}}{10}$

2 (a) $\frac{\sqrt{5}}{10}$ (b) $\frac{\sqrt{2}}{10}$ (c) $\frac{5\sqrt{3}}{3}$

(d) $\frac{7\sqrt{2}}{6}$ (e) $\frac{2\sqrt{3}}{15}$

3 (a) $\frac{3\sqrt{2}}{2}$ (b) $\frac{\sqrt{15}}{3}$ (c) $\frac{3\sqrt{10}}{10}$

(d) $\frac{\sqrt{3}}{3}$ (e) $\frac{\sqrt{15}}{5}$

4 (a) 22 (b) 47 (c) -11
(d) -1 (e) 2 (f) $a - b$

5 (a) $\sqrt{2} + 1$ (b) $\sqrt{3} + 1$ (c) $2\sqrt{5} - 2$
(d) $9 + 3\sqrt{2}$ (e) $\sqrt{3} + \sqrt{2}$ (f) $\sqrt{7} - \sqrt{5}$
(g) $\sqrt{15} + 2\sqrt{3} - \sqrt{5} - 2$
(h) $8 - 4\sqrt{3} + 2\sqrt{5} - \sqrt{15}$

Exercise 14

1. (a) 30, 150 (b) 45, 315 (c) 41, 221
   (d) 20, 340 (e) 52, 232 (f) 35, 145
   (g) 82, 98 (h) 81, 261 (i) 67, 293
   (j) 10, 350 (k) 60, 120 (l) 60, 240.
2. (a) 60, 300 (b) 53·1 or 126·9 (c) 35·0, 215·0
   (d) 19·5, 160·5 (e) 33·6, 326·4 (f) 59·0, 239·0.
3. (a) 210, 330 (b) 135, 225 (c) 159, 339
   (d) 107, 253 (e) 135, 315 (f) 240, 300
   (g) 104, 284 (h) 190, 350 (i) 158, 202.
4. (a) 194·5, 345·5 (b) 126·9, 233·1 (c) 161·6, 341·6
   (d) 151·0, 209·0 (e) 228·6, 311·4 (f) 122·0, 302·0.
5. (a) 18·8, 161·2 (b) 154·2, 205·8 (c) 34·1, 214·1
   (d) 75·5, 284·5 (e) 225, 315 (f) 116·6, 296·6
   (g) 36·9, 143·1 (h) 96·4, 263·6 (i) 32·0, 212·0
   (j) 218·7, 321·3 (k) 120, 240 (l) 11·3, 191·3
   (m) 58·2, 121·8 (n) 53·1, 306·9 (o) 138·8, 318·8
   (p) 41·8, 138·2 (q) 33·6, 326·4 (r) 150·3, 330·3
   (s) 199·5, 340·5°.